Wide-Band Polarizer in Circular Waveguide Loaded with Dielectric Discs

P. J. MEIER, MEMBER, IEEE, AND S. ARNOW, MEMBER, IEEE

Abstract—A converter between linear and circular polarization has been designed in circular waveguide by the use of periodic loading with oblong dielectric discs. The disc loading increases the single-mode (TE-11) bandwidth relative to that of homogeneously filled circular waveguide. Moreover, periodic loading gives the designer freedom to adjust the disc thickness and shape to meet certain requirements. With the abutting disc-loaded waveguide as a reference, the dielectric loading is increased in one plane of polarization and decreased in the orthogonal plane. In a wide-band design, this differential loading gives nearly constant 90° difference in time phase between space-quadrature component waves, the condition for circular polarization.

Such a polarizer has been constructed with four oblong discs. Across a frequency bandwidth of about 15 percent, the measured phase difference is within 2° of the desired value and the SWR is within 1.5 dB.

I. Introduction

IT IS WELL KNOWN that a conversion between linear and circular polarization can be accomplished in circular waveguide by providing differential loading for two crossed TE-11 modes. Examples of this technique include flat dielectric slabs [1], [2], as well as a dielectric rod with flattened sides [3]. Unfortunately, these configurations lower the cutoff frequency of the next higher mode.

Whenever it is possible, waveguides are used at frequencies where only one mode, the dominant mode, can propagate. This places all higher-order modes beyond cutoff, thereby avoiding spurious resonances and other harmful effects. Thus, the spacing between the cutoff frequencies of the dominant and the next higher mode determines the useful bandwidth of a waveguide. It has been shown that the useful bandwidth of circular waveguide can be greatly increased by periodic loading with dielectric discs [4]. This wider bandwidth is useful in the design of many components, such as the polarization converter to be described.

Differential loading can be achieved in disc-loaded waveguide by oblong discs which present different values of "equivalent dielectric constant" in the two orthogonal planes of polarization. Such a configuration does not proportionately load the next higher mode. Moreover, the loading may be increased in one plane and decreased in the orthogonal plane, thus minimizing the departure from the "normal" loading in the abutting waveguide. Impedance matching can be obtained by shaping the discs, to taper the element strength from the center to the ends of the polarizer.

Manuscript received May 13, 1965; revised June 25, 1965. This paper was presented at the 1965 IEEE MTT Symp., Clearwater, Fla.
S. Arnow is with Wheeler Labs. Inc., Great Neck, N. Y. P. J. Meier is with Loral Electronics Systems, New York, N. Y.

After a brief review of the modes which propagate in disc-loaded waveguide, formulas will be presented which explain how wide-band polarizer operation is obtained. Methods of physically realizing the desired differential loading will be discussed, and a program that led to the design of a working model will be outlined. It will be shown that this model exhibited excellent conversion of polarization across a wide band, as predicted by theory.

II. Symbols

f = frequency
f_c = cutoff frequency of dielectric loaded waveguide
f_{oc} = cutoff frequency of unloaded (empty) waveguide
λ = free-space wavelength
λ_m = free-space wavelength at midband
λ_c = cutoff wavelength (1.706 diameters, for dominant mode in circular waveguide)
$u = \lambda/\lambda_c = f_{oc}/f = \sqrt{k_e}f_c/f$
k = dielectric constant
k_d = dielectric constant of loading disc
k_e = equivalent dielectric constant
$k_1 = k_e$ in waveguide abutting polarizer
$k_2 = k_e$ in high-k polarizer channel
$k_3 = k_e$ in low-k polarizer channel
t = thickness of dielectric disc
d = spacing of discs on centers
β = phase retardation per unit length
λ_2, $\beta_3 = \beta$ in respective polarizer channels
ϕ = phase difference between polarizer channels
L = length of polarizer
u_0, f_0, ϕ_0 = quantities for zero slope of phase difference.

III. Modes in Disc-Loaded Waveguide

Consider a circular waveguide, periodically loaded with dielectric discs, as shown in Fig. 1. These discs strongly load the dominant (TE-11) mode, but have little effect on the (TM-01) mode which normally limits the single-mode bandwidth. Figure 2 compares the cutoff frequencies of the first several modes for both uniform and periodic loading. Here the frequency f has been normalized with respect to the lowest cutoff frequency f_c. The most striking difference between the two cases is the separation of the TE-11 and TM-01 modes. For our example, using alumina discs that fill one third the

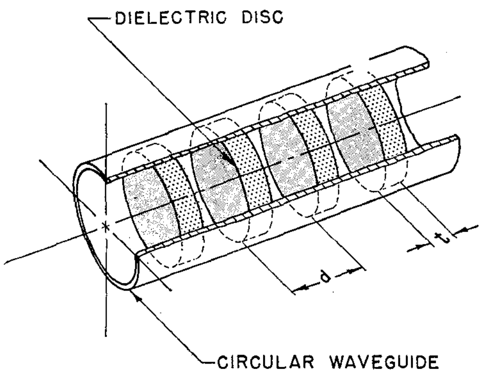

Fig. 1. Dielectric disc-loaded circular waveguide.

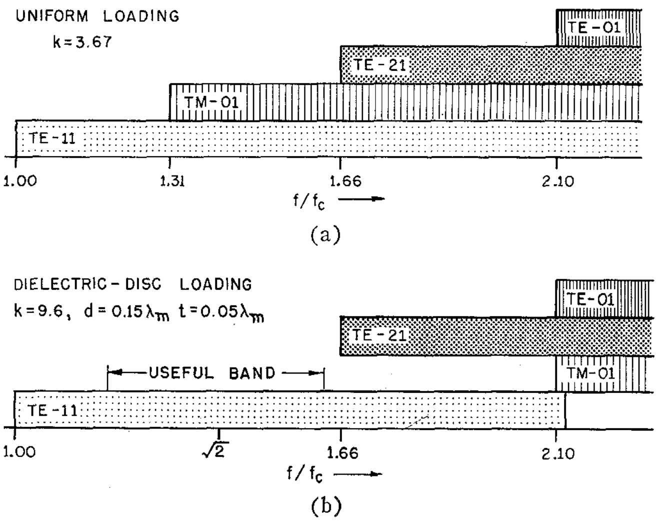

Fig. 2. Relative cutoff frequencies of several modes. (a) Modes propagating in uniformly loaded circular waveguide. (b) Modes propagating in disc loaded circular waveguide.

volume of the waveguide, the TM-01 cutoff frequency is so high that it no longer limits the useful bandwidth. The TE-21 mode then becomes the principal limitation. Note also in Fig. 2 that the passband for the TE-11 mode is finite for disc loading, and semi-infinite for uniform loading. Although the periodic structure does have additional stopbands, these will be at frequencies far above the useful band, provided the disc spacing is small with respect to the wavelength. Thus, in the useful band, we can treat the disc-loaded waveguide as a metal pipe of the same diameter, completely filled with some equivalent dielectric constant k_e, given by

$$k_e \simeq 1 + (k_d - 1)\frac{t}{d} \tag{1}$$

where k_d is the dielectric constant of the disc, and t and d are the disc thickness and spacing as shown in Fig. 1. Although (1) is derived from the static-field equivalent circuits of [4], it is accurate within several percent for the dimensions and frequencies under consideration here. With circular discs, k_e is the same in all planes of polarization. The shape of the disc, however, can be modified to obtain different values of k_e in orthogonal planes, thus enabling the formation of a polarizer.

IV. Theory of Polarizer Operation

An exact theoretical analysis of a circular waveguide, periodically loaded with discs of arbitrary shape, is a formidable task. To reveal the mechanism by which wide-band performance is obtained, we shall make some simplifying assumptions. Using the concept of "equivalent dielectric constant," we may write an expression for the phase lag per unit length as

$$\beta = \frac{2\pi}{\lambda}\sqrt{k_e - u^2}, \tag{2}$$

where λ is the free-space wavelength and u is λ/λ_c, the ratio of free space over cutoff wavelength.

Figure 3 is a plot of the phase shift through a given length L for three different values of k_e. Let k_1 designate the value of k_e in the waveguide abutting the polarizer, k_2 the k_e in the linear polarization of greatest loading, and k_3 the k_e in the linear polarization of least loading. Each curve rises sharply, levels off, and asymptotically approaches a straight line whose slope is proportional to the square root of k_e. A wide band exists, over which the curves are approximately parallel. If a wave is launched with equal components in each of the principal planes (or channels) of the polarizer, a nearly constant 90° difference in time phase can be achieved in propagation through the polarizer. This can be shown quantitatively by writing the equation for the phase difference between components

$$\phi = L(\beta_2 - \beta_3) \tag{3}$$

where β_2 and β_3 are the phase constants in the high-k and low-k channels, respectively. In terms of the equivalent dielectric constants,

$$\phi = \frac{2\pi L}{\lambda}\left[\sqrt{k_2 - u^2} - \sqrt{k_3 - u^2}\right.$$

or

$$\phi = \frac{2\pi L}{c}\left[\sqrt{f^2 k_2 - f_{oc}^2} - \sqrt{f^2 k_3 - f_{oc}^2}\right] \tag{4}$$

where c is the velocity of light in free space, and f_{oc} is the cutoff frequency with no dielectric loading.

Taking the derivative of the above equation with respect to frequency, we find

$$\frac{d\phi}{df} = \frac{2\pi L}{c}\left[\frac{k_2}{\sqrt{k_2 - u^2}} - \frac{k_3}{\sqrt{k_3 - u^2}}\right]. \tag{5}$$

Setting (5) equal to zero, we may solve for u_0, the inverse frequency ratio at which there is zero slope of phase difference:

$$u_0 = \sqrt{\frac{k_2 k_3}{k_2 + k_3}}. \tag{6}$$

Thus, the frequency at which $d\phi/df = 0$ is

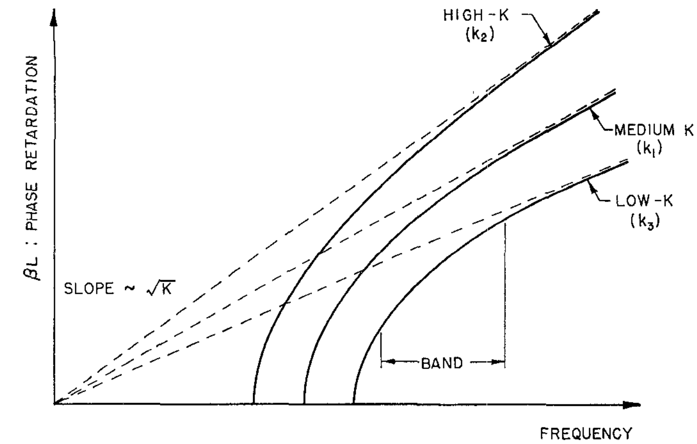

Fig. 3. Phase vs. frequency for differential loading.

$$f_o = f_{oc}\sqrt{\frac{k_2 + k_3}{k_2 k_3}}. \tag{7}$$

The phase difference at f_0 is found by substituting in (4)

$$\phi_0 = \frac{2\pi L}{\lambda_c} \frac{k_2 - k_3}{\sqrt{k_2 k_3}}. \tag{8}$$

The slope of the phase difference at high frequencies is determined by taking the limit of (5) as u approaches zero:

$$\lim_{u\to 0} \frac{d\phi}{df} = \frac{2\pi L}{c}(\sqrt{k_2} - \sqrt{k_3}) = \frac{2\pi L}{c} \frac{k_2 - k_3}{\sqrt{k_2} + \sqrt{k_3}}. \tag{9}$$

Figure 4 summarizes these relations, and shows other critical points, in a plot of the phase difference between channels vs. frequency. Note that the three critical frequencies labeled in Fig. 4 are related by

$$\sqrt{2}\frac{f_{oc}}{\sqrt{k_2}} < f_{oc}\sqrt{\frac{k_2 + k_3}{k_2 k_3}} < \sqrt{2}\frac{f_{oc}}{\sqrt{k_3}}. \tag{10}$$

To minimize the departure from the "normal" loading in the waveguide which abuts the polarizer, we will set $f_{oc}/\sqrt{k_1}$ midway between $f_{oc}/\sqrt{k_2}$ and $f_{oc}/\sqrt{k_3}$. This requires

$$\frac{f_o}{f_{oc}} = \sqrt{\frac{2}{k_1}}. \tag{11}$$

It should be remembered that f_0 has been defined simply as the frequency at which the phase slope is zero. No practical restrictions have yet been placed upon f_o. Now if we set f_o equal to our midband operating frequency, we require that

$$\frac{f_o}{f_c} = \sqrt{2}$$

in the waveguide which abuts the polarizer. Returning to Fig. 2, we see that this is not advisable for uniformly filled circular waveguide, since a spurious mode could then propagate. Disc-loaded waveguide, however, is well qualified to fill our needs: a 30 percent band centered at $f_o/f_c = \sqrt{2}$ is available. This band clears the lowest cutoff by 20 percent and avoids spurious modes.

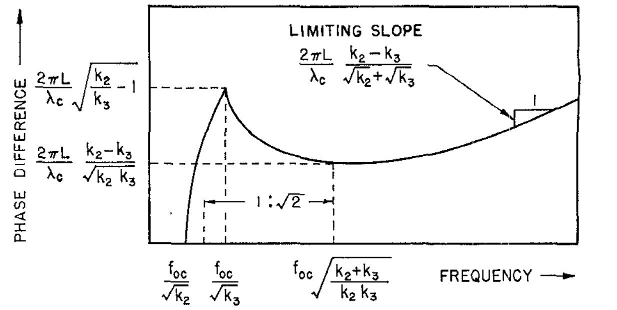

Fig. 4. Frequency variation of phase difference between channels.

Now that we have specified the frequency ratio at midband, we may set (6) equal to (11) and show that

$$k_1 = \frac{2k_2 k_3}{k_2 + k_3}. \tag{12}$$

Now we may fix the midband phase shift at 90° by setting (8) equal to $\pi/2$:

$$\frac{k_2 - k_3}{\sqrt{k_2 k_3}} = \frac{\lambda_c}{4L}. \tag{13}$$

By solving (12) and (13) simultaneously, we find

$$k_2 \text{ or } k_3 = k_1 \frac{\sqrt{\left(\frac{8L}{\lambda_c}\right)^2 + 1}}{\sqrt{\left(\frac{8L}{\lambda_c}\right)^2 + 1} \pm 1}. \tag{14}$$

Thus, for any polarizer length we may select, (14) may be solved to find k_2 and k_3. Just how these values of equivalent k may be physically realized will be our next topic.

V. Physical Realization

The desired differential loading can be achieved by any of a variety of disc shapes. Figure 5 shows some of the shapes that were studied during the course of the program. Several factors were considered in choosing the shape for a test model. These factors included ease of fabrication, ability to obtain sufficient differences between k_2 and k_3, peak power capacity, and spurious-mode suppression. For machining ease, the shapes of Fig. 5(a) and (b) were preferred. Experiments showed that all shapes except Fig. 5(b) exhibited sufficient differences between k_2 and k_3. As for power capacity, Fig. 5(d) was preferred for the right angle where the dielectric edge leaves the metal wall. For single-mode operation, a shape must be selected that does not strongly load the TE-21 mode. The shape of Fig. 5(a) appears best in this respect.

After weighing these factors, the oblong shape shown

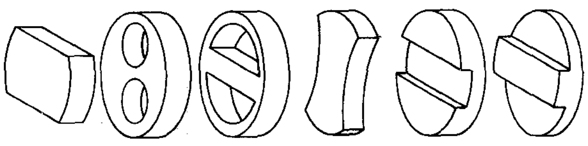

Fig. 5. Alternate configurations for shaped discs.

in Fig. 5(a) was selected. This simple shape could be easily described and modified for experiments. When the electric field is parallel to the flat edge of such a disc, the equivalent k is high because the thickness is greater than normal, and series air gaps are avoided. For the orthogonal polarization, series air gaps more than compensate for the thickness increase, and the net result is a low equivalent k.

VI. Design of the Developmental Model

The relationships given in Section IV were used to predict the performance of polarizers of various lengths. At first, the spacing between the centers of polarizer discs was taken as 0.15 λ_0, the same spacing as in the waveguide which abuts the polarizer. Sintered alumina of high density and purity, with a dielectric constant of 9.6, was chosen as the disc material. Calculations showed that three or four polarizer discs would be adequate for a 15 percent frequency band, which was approximately the design objective.

Since an exact theoretical solution would be difficult, an experimental program was launched to find the optimum dimensions for the developmental model. A model containing four identical polarizing discs was constructed, and its equivalent circuit was determined for each of two orthogonal linear polarizations. For test purposes, the linear polarizations were aligned with the high-k and low-k planes. Thus, there was no coupling to the crossed mode, and the network could be analyzed as a separate two-port for each polarization. Figure 6 shows the test setup and the equivalent circuit thereby evaluated. When the polarizer is match-terminated, slotted line measurements reveal the circuit parameters l_1 and n. Next, a short circuit is placed at the end of the polarizer, and l_2 is determined. Polarizer dimensions are then varied to obtain experimentally the two ideal conditions: no reflection ($n=1$), and the path length (l_1+l_2) having a difference of 90° for orthogonal polarization over the bandwidth.

The dimensions of each of the four shaped discs were then varied by like amounts to empirically determine the performance capability. Although four identical elements showed nearly constant phase difference, the standing-wave ratio (SWR) could not be held lower than 4 dB. The next step was to "taper" the element strength, that is, to increase the polarizing strength of the two center elements while decreasing the strength of the two outer elements. Final corrections were obtained by a slight variation of the interelement spacings. Dimensions of the developmental polarizer are given in Fig. 7; Fig. 8 is a photograph of this model.

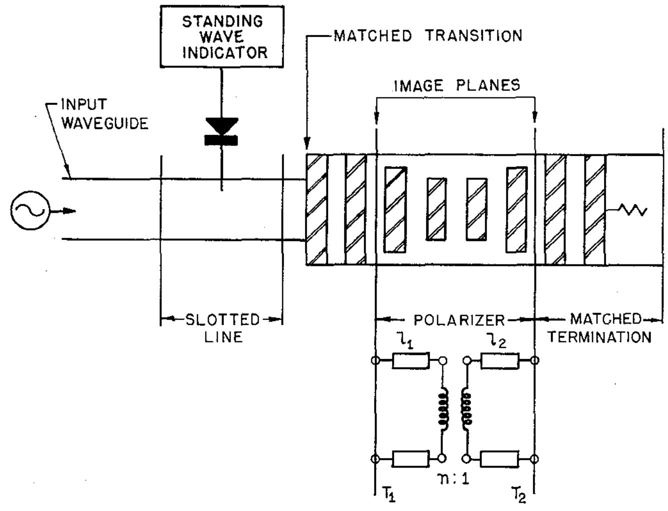

Fig. 6. Definition and measurement of polarizer characteristics.

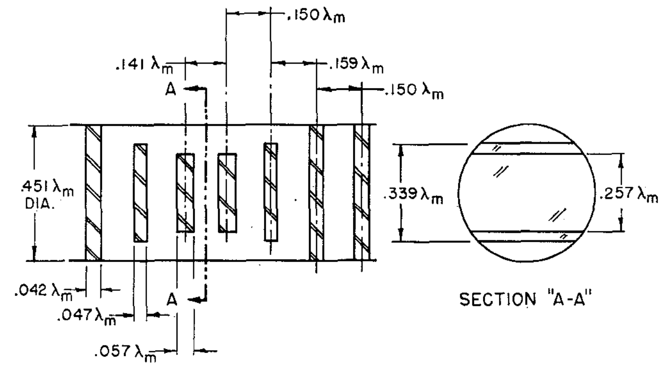

Fig. 7. Dimensions of developmental polarizer.

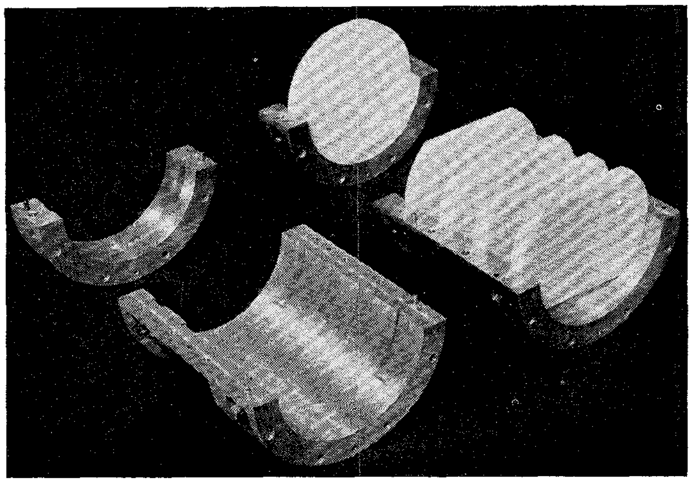

Fig. 8. Photograph of polarizer.

VII. Performance of the Developmental Model

Next we shall discuss the reflection, phase characteristics, axial ratio, mode purity, and power capacity of the developmental model. The reflection, measured in each of the polarizer's principal planes, is plotted in Fig. 9. Note that the SWR is within 1.5 dB across a bandwidth of about 15 percent. Figure 10 shows the phase difference between channels over the same band-

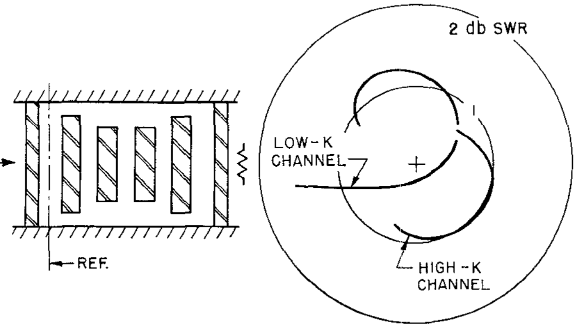

Fig. 9. Measured reflection over 15 percent band.

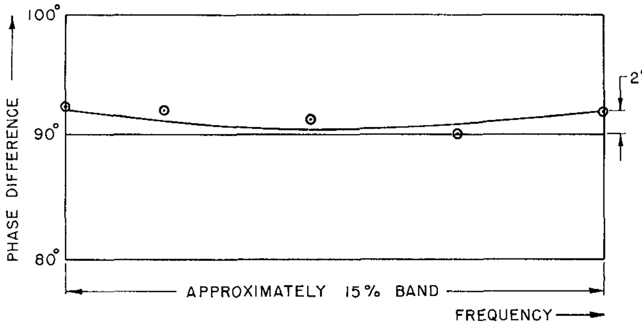

Fig. 10. Measured phase difference between channels.

width; the variation is within $\pm 1°$ from the average, and within 2° of the design objective (90°). From these properties, the axial ratio is computed to be within 0.3 dB across the 15 percent frequency band.

As we saw earlier, the TE-21 cutoff frequency imposes the upper limit on the useful bandwidth of disc-loaded waveguide. One might wonder if the thicker discs in the polarizer region would load the TE-21 mode enough to allow it to resonate in, or too near, the band. A cavity test was performed to resolve this question. The polarizer was short-circuited at both input and output ports, and crossed electric probes were inserted into the resultant cavity to excite the TE-21 mode strongly. The TE-21 cutoff, calculated from the lowest cavity resonance, occurred above the operating band. This demonstrated that the dielectric material removed from the periphery of the oblong discs more than compensated for the increased TE-21 loading provided by the thickened discs.

Experiments were conducted to determine the capacity of the developmental model to transmit high peak power. The power capacity of the test model was limited by several factors. First, no attempt was made to bond the edge of the discs to the metal waveguide walls. Thus high gradients appeared in the air gaps between the high-k discs and the wall. Also, the simple oblong shapes encouraged arcing at the sharp edges and corners of the disc. With these handicaps, the power capacity of the model is only about 3 percent that of the largest single-mode air-filled (44 ohm) coaxial cable that could be used for the same band. With medium-k discs shaped as in Fig. 5(d), and with air gaps eliminated, it is estimated that the power capacity would be 20 percent that of the same coaxial cable.

VIII. Conclusions

A circular waveguide, loaded with oblong dielectric discs, has been analyzed in terms of a uniform line, with different values of "equivalent dielectric constant" for orthogonal polarizations. This analysis shows that excellent conversion between linear and circular polarization can be obtained across a wide frequency band. This theory has been confirmed by constructing a four-disc polarizer, which exhibited a 0.3-dB axial ratio across a 15 percent band. It is estimated that a 30 percent band can be accommodated, thus realizing the full bandwidth of disc-loaded waveguide. The power capacity of the developmental model is modest, but useful for some transmitter applications. Higher power capacity may be achieved by lower-k discs of improved shape, bonded to the waveguide walls.

Acknowledgment

This development has been conducted for the Bell Telephone Laboratories. The direction and encouragement provided by R. L. Mattingly, P. H. Smith and others at BTL are gratefully acknowledged.

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